Sequence Alignment & Computational Thinking Michael Schatz

Aug 29, 2012 QB Bootcamp Lecture 2





Outline

Part I: Overview & Fundamentals

Part 2: Sequence Analysis Theory

- Intro to alignment and algorithms
- Understanding Bowtie

Part 3: Genomics Resources

Part 4: Example Analysis

Advances in Sequencing: Now Generation Sequencing



Illumina HiSeq 2000 Sequencing by Synthesis

> >60Gbp / day 100bp reads



PacBio SMRT-sequencing

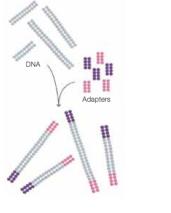
> ~IGbp / day Long Reads



Oxford Nanopore Nanopore sensing

Many GB / day? Very Long Reads?

Illumina Sequencing by Synthesis



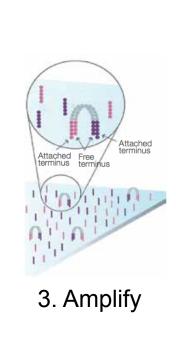
1. Prepare

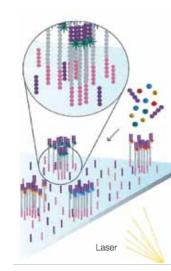
2. Attach

Adapter

DNA fragment

Dense lawn of primers





4. Image









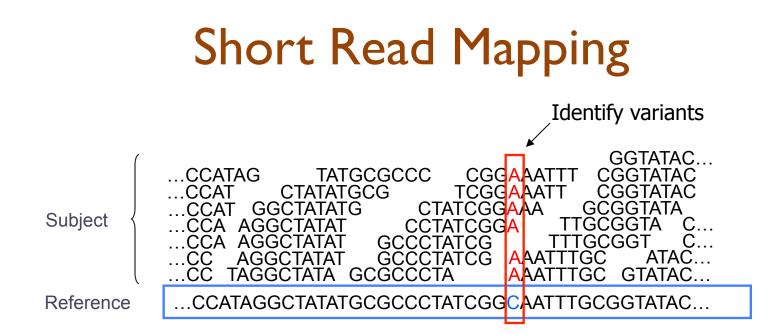


5. Basecall

Sequence Alignment

- A very common problem in computational biology is to find occurrences of one sequence in another sequence
 - Genome Assembly
 - Gene Finding
 - Comparative Genomics
 - Functional analysis of proteins
 - Motif discovery
 - SNP analysis
 - Phylogenetic analysis
 - Primer Design
 - Personal Genomics

- ...



- Given a reference and many subject reads, report one or more "good" end-toend alignments per alignable read
 - Fundamental computation to genotyping and many assays
 - RNA-seq
 ChIP-seq
 Dnase-seq
 Hi-C-seq
- Desperate need for scalable solutions
 - Single human requires >1,000 CPU hours / genome
 - 1000 hours * 1000 genomes = IM CPU hours / project

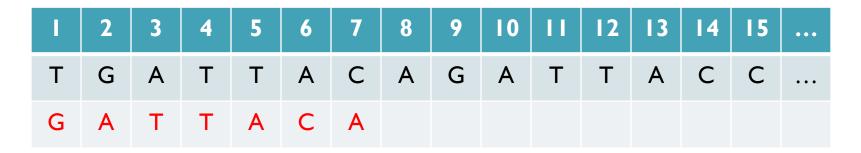
Computational Biology

"Computer science is no more about computers than astronomy is about telescopes." Edsger Dijkstra

- Computer Science = Science of Computation
 - Solving problems, designing & building systems
 - Computers are very, very dumb, but we can instruct them
 - Build complex systems out of simple components
 - They will perfectly execute instructions forever
- CompBio = Thinking Computationally about Biology
 - Processing: Make more powerful instruments, analyze results
 - Designing & Understanding: protocols, procedures, systems
- Sequence Alignment
 - I. Brute Force
 - 2. Suffix Arrays
 - 3. Inexact Alignment
 - 4. Bowtie

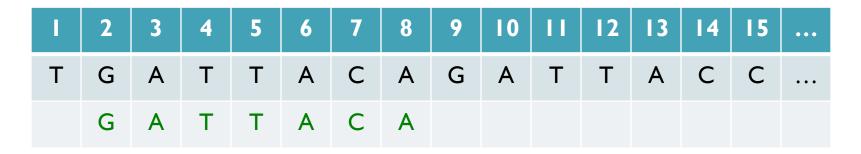
- Computational Thinking
 - I. Algorithm
 - 2. Data structure
 - 3. Computational Analysis
 - 4. Computational Modeling

- Where is GATTACA in the human genome?
- Strategy I: Brute Force



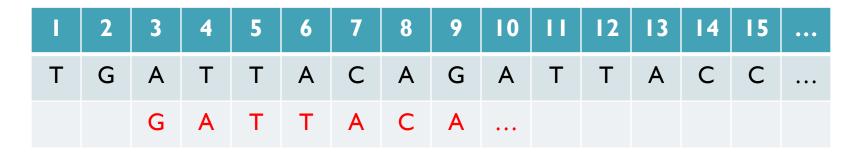
No match at offset I

- Where is GATTACA in the human genome?
- Strategy I: Brute Force



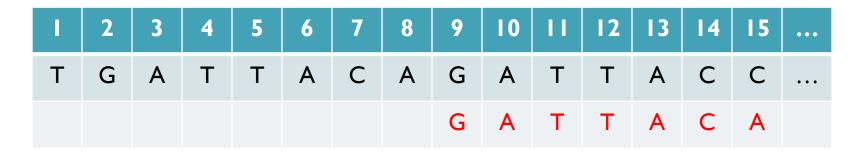
Match at offset 2

- Where is GATTACA in the human genome?
- Strategy I: Brute Force



No match at offset 3...

- Where is GATTACA in the human genome?
- Strategy I: Brute Force



No match at offset 9 <- Checking each possible position takes time

Brute Force Analysis

- Brute Force:
 - At every possible offset in the genome:
 - Do all of the characters of the query match?
- Analysis
 - Simple, easy to understand
 - Genome length = n
 - Query length = m
 - Comparisons: (n-m+1) * m
- Overall runtime: O(nm)

[How long would it take if we double the genome size, read length?] [How long would it take if we double both?]

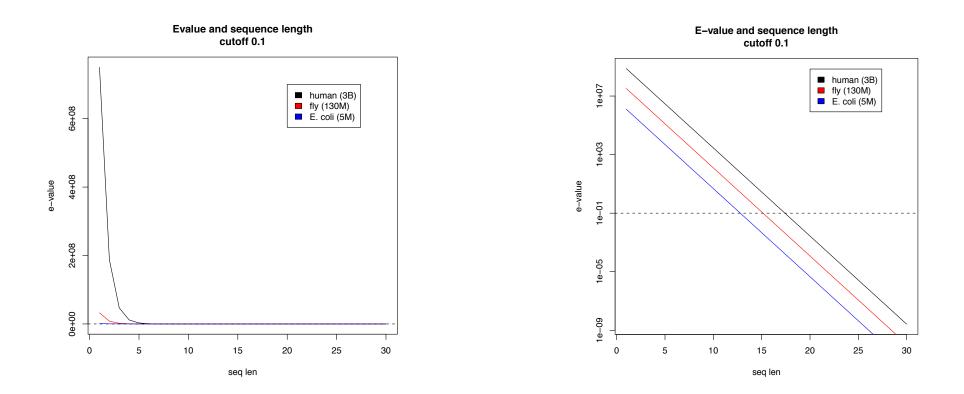
[3B] [7] [21B]

Expected Occurrences

The expected number of occurrences (e-value) of a given sequence in a genome depends on the length of the genome and inversely on the length of the sequence

- I in 4 bases are G, I in 16 positions are GA, I in 64 positions are GAT, ...
- I in 16,384 should be GATTACA
- $E=n/(4^{m})$

[183,105 expected occurrences] [How long do the reads need to be for a significant match?]



Brute Force Reflections

Why check every position?

- GATTACA can't possibly start at position 15

[WHY?]



- Improve runtime to O(n + m)

[3B + 7]

- If we double both, it just takes twice as long
- Knuth-Morris-Pratt, 1977
- Boyer-Moyer, 1977, 1991
- For one-off scans, this is the best we can do (optimal performance)
 - We have to read every character of the genome, and every character of the query
 - For short queries, runtime is dominated by the length of the genome

Suffix Arrays: Searching the Phone Book

- What if we need to check many queries?
 - We don't need to check every page of the phone book to find 'Schatz'
 - Sorting alphabetically lets us immediately skip 96% (25/26) of the book without any loss in accuracy
- Sorting the genome: Suffix Array (Manber & Myers, 1991)
 - Sort every suffix of the genome



Split into n suffixes Sort suffixes alphabetically

[Challenge Question: How else could we split the genome?]

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = I; Hi = 15;

Lo	#	Sequence	Pos
->	-	ACAGATTACC	6
	2	ACC	13
	3	AGATTACC	8
	4	ATTACAGATTACC	3
	5	ATTACC	10
	6	C	15
	7	CAGATTACC	7
	8	CC	14
	9	GATTACAGATTACC	2
	10	GATTACC	9
	11	TACAGATTACC	5
	12	TACC	12
	13	TGATTACAGATTACC	I
	14	TTACAGATTACC	4
Hi	15	TTACC	11

- Strategy 2: Binary search ۲
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA ٠
 - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
 - Middle = Suffix[8] = CC

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 => Higher: Lo = Mid + I
 - Lo = 9; Hi = 15;

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 - Middle = Suffix[8] = CC
 => Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC

	#	Sequence	Pos
	I	ACAGATTACC	6
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 - Compare to the middle, refine as higher or lower
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 - Middle = Suffix[8] = CC = Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC => Lower: Hi = Mid - I
 - Lo = 9; Hi = 11;

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 => Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC
 => Lower: Hi = Mid 1
 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC

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	2	ACC	13
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 => Lower: Hi = Mid I
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15	TTACC	

Lo

Hi

- Strategy 2: Binary search
 - Compare to the middle, refine as higher or lower
- Searching for GATTACA
 - Lo = 1; Hi = 15; Mid = (1+15)/2 = 8
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 => Higher: Lo = Mid + I
 - Lo = 9; Hi = 15; Mid = (9+15)/2 = 12
 - Middle = Suffix[12] = TACC
 => Lower: Hi = Mid 1
 - Lo = 9; Hi = 11; Mid = (9+11)/2 = 10
 - Middle = Suffix[10] = GATTACC
 => Lower: Hi = Mid I
 - Lo = 9; Hi = 9; Mid = (9+9)/2 = 9
 - Middle = Suffix[9] = GATTACA...
 => Match at position 2!

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	13	TGATTACAGATTACC	I
	14	TTACAGATTACC	4
	15	TTACC	11

Binary Search Analysis

Binary Search

Initialize search range to entire list mid = (hi+lo)/2; middle = suffix[mid] if query matches middle: done else if query < middle: pick low range else if query > middle: pick hi range Repeat until done or empty range

[WHEN?]

- Analysis
 - More complicated method
 - How many times do we repeat?
 - How many times can it cut the range in half?
 - Find smallest x such that: $n/(2^x) \le I$; $x = lg_2(n)$ [32]
- Total Runtime: O(m lg n)
 - More complicated, but much faster!
 - Looking up a query loops 32 times instead of 3B

[How long does it take to search 6B or 24B nucleotides?]

Suffix Array Construction

 How can we store the suffix array? [How many characters are in all suffixes combined?]

$$S = 1 + 2 + 3 + \dots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = O(n^2)$$

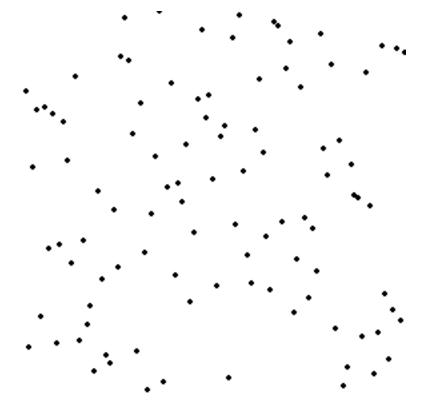
- Hopeless to explicitly store 4.5 billion billion characters
- Instead use implicit representation
 - Keep I copy of the genome, and a list of sorted offsets
 - Storing 3 billion offsets fits on a server (12GB)
- Searching the array is very fast, but it takes time to construct
 - This time will be amortized over many, many searches
 - Run it once "overnight" and save it away for all future queries

Sorting

Quickly sort these numbers into ascending order: 14, 29, 6, 31, 39, 64, 78, 50, 13, 63, 61, 19

[How do you do it?]

6, 14, 29, 31, 39, 64, 78, 50, 13, 63, 61, 19 6, 13, 14, 29, 31, 39, 64, 78, 50, 63, 61, 19 6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61 6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61 6, 13, 14, 19, 29, 31, 39, 64, 78, 50, 63, 61 6, 13, 14, 19, 29, 31, 39, 50, 64, 78, 63, 61 6, 13, 14, 19, 29, 31, 39, 50, 61, 64, 78, 63 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78 6, 13, 14, 19, 29, 31, 39, 50, 61, 63, 64, 78



http://en.wikipedia.org/wiki/Selection_sort

Selection Sort Analysis

• Selection Sort (Input: list of n numbers)

```
for pos = I to n
    // find the smallest element in [pos, n]
    smallest = pos
    for check = pos+I to n
        if (list[check] < list[smallest]): smallest = check</pre>
```

// move the smallest element to the front tmp = list[smallest] list[pos] = list[smallest] list[smallest] = tmp

• Analysis

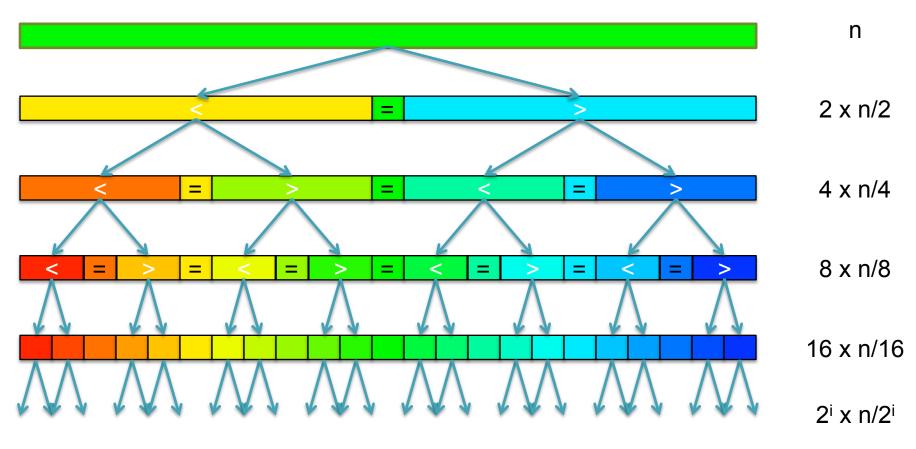
$$T = n + (n - 1) + (n - 2) + \dots + 3 + 2 + 1 = \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} = O(n^2)$$

- Outer loop: pos = I to n
- Inner loop: check = pos to n
- Running time: Outer * Inner = $O(n^2)$ [4.5 Billion Billion]

[Challenge Questions: Why is this slow? / Can we sort any faster?]

Divide and Conquer

- Selection sort is slow because it rescans the entire list for each element
 - How can we split up the unsorted list into independent ranges?
 - Hint I: Binary search splits up the problem into 2 independent ranges (hi/lo)
 - Hint 2: Assume we know the median value of a list

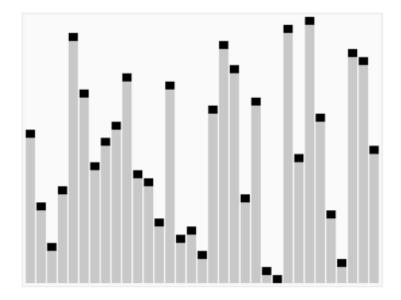


[How many times can we split a list in half?]

QuickSort Analysis

QuickSort(Input: list of n numbers)
 // see if we can quit
 if (length(list)) <= 1): return list

```
// split list into lo & hi
pivot = median(list)
lo = {}; hi = {};
for (i = I to length(list))
        if (list[i] < pivot): append(lo, list[i])
        else: append(hi, list[i])</pre>
```



http://en.wikipedia.org/wiki/Quicksort

// recurse on sublists
return (append(QuickSort(lo), QuickSort(hi))

• Analysis (Assume we can find the median in O(n))

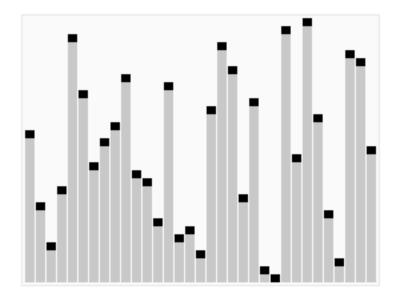
$$T(n) = \begin{cases} O(1) & \text{if } n \le 1\\ O(n) + 2T(n/2) & \text{else} \end{cases}$$

$$T(n) = n + 2(\frac{n}{2}) + 4(\frac{n}{4}) + \dots + n(\frac{n}{n}) = \sum_{i=0}^{lg(n)} \frac{2^{i}n}{2^{i}} = \sum_{i=0}^{lg(n)} n = O(n\lg n) \quad \text{[~94B]}$$

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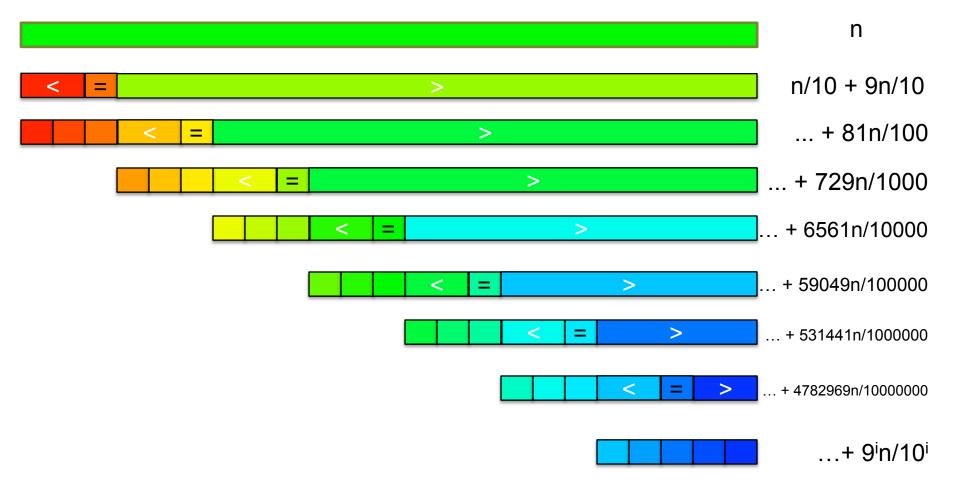
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Picking the Median

• What if we miss the median and do a 90/10 split instead?



[How many times can we cut 10% off a list?]

Randomized Quicksort

- 90/10 split runtime analysis $T(n) = n + T(\frac{n}{10}) + T(\frac{9n}{10})$ $T(n) = n + \frac{n}{10} + T(\frac{n}{100}) + T(\frac{9n}{10}) + \frac{9n}{10} + T(\frac{9n}{100}) + T(\frac{81n}{100})$ $T(n) = n + n + T(\frac{n}{100}) + 2T(\frac{9n}{100}) + T(\frac{81n}{100})$ $T(n) = \sum_{i=0}^{\log_{10/9}(n)} n = O(n \lg n)$ Find smallest x s.t. (9/10)^x n \le 1 $(10/9)^{x} \ge n$ $x \ge \log_{10/9} n$
- If we randomly pick a pivot, we will get at least a 90/10 split with very high probability
 - Everything is okay as long as we always slice off a fraction of the list

[Challenge Question: What happens if we slice I element]



THE G-NOME PROJECT

Break

Recap

• Programming

- I. Variables & Arguments
- 2. Conditionals
- 3. Loops
- 4. Functions
- Sequence Alignment
 - I. Brute Force Exact Matching simple but slow
 - 2. Suffix Arrays very fast matching
 - 3. Inexact Alignment TODO
 - 4. Bowtie TODO
- Computational Thinking
 - I. Algorithm "Formal" recipe, precise definition of problem
 - 2. Data structure Choices of how to represent data
 - 3. Computational Analysis Time, space requirements
 - 4. Computational Modeling Characterize expected results

In-exact alignment

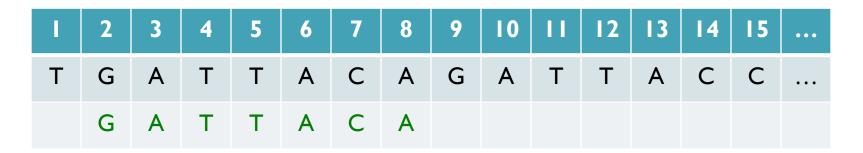
- Where is GATTACA *approximately* in the human genome?
 - And how do we efficiently find them?
- It depends...
 - Define 'approximately'
 - Hamming Distance, Edit distance, or Sequence Similarity
 - Ungapped vs Gapped vs Affine Gaps
 - Global vs Local
 - All positions or the single 'best'?
 - Efficiency depends on the data characteristics & goals
 - Smith-Waterman: Exhaustive search for optimal alignments
 - BLAST: Hash-table based homology searches
 - Bowtie: BWT alignment for short read mapping

• Where is GATTACA *approximately* in the human genome?



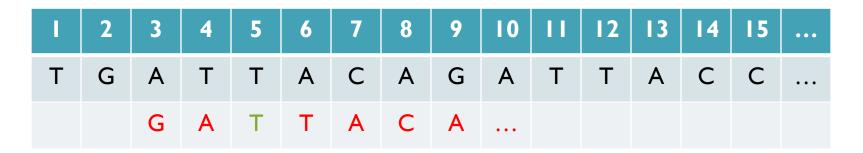
Match Score: 1/7

• Where is GATTACA *approximately* in the human genome?



Match Score: 7/7

• Where is GATTACA *approximately* in the human genome?



Match Score: 1/7

• Where is GATTACA *approximately* in the human genome?



Match Score: 6/7 <- We may be very interested in these imperfect matches Especially if there are no perfect end-to-end matches

Hamming Distance

- How many characters are different between the 2 strings?
 Minimum number of substitutions required to change transform A into B
- Traditionally defined for end-to-end comparisons

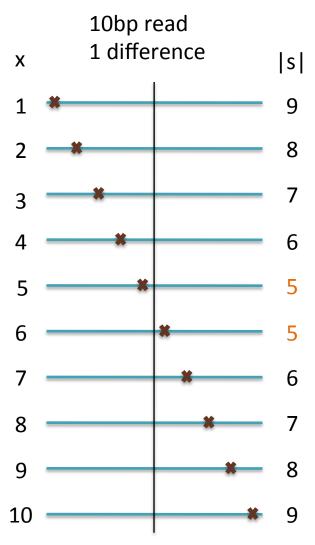
 Here end-to-end (global) for query, partial (local) for reference

- Find all occurrences of GATTACA with Hamming Distance $\leq I$
- Find all occurrences with minimal Hamming Distance [What is the running time of a brute force approach?]

Seed-and-Extend Alignment

Theorem: An alignment of a sequence of length mwith at most k differences **must** contain an exact match at least s=m/(k+1) bp long (*Baeza-Yates* and Perleberg, 1996)

- Proof: Pigeonhole principle
 - I pigeon can't fill 2 holes
- Seed-and-extend search
 - Use an index to rapidly find short exact alignments to seed longer in-exact alignments
 - BLAST, MUMmer, Bowtie, BWA, SOAP, ...
 - Specificity of the depends on seed length
 - Guaranteed sensitivity for k differences
 - Also finds some (but not all) lower quality alignments <- heuristic

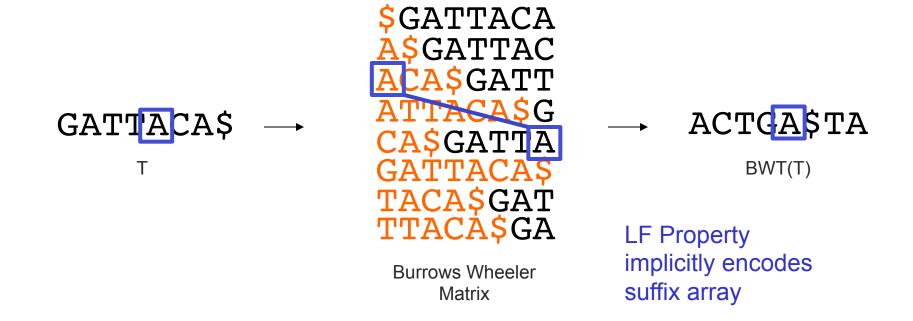




Bowtie: Ultrafast and memory efficient alignment of short DNA sequences to the human genome

Slides Courtesy of Ben Langmead (langmead@umiacs.umd.edu)

Burrows-Wheeler Transform



- Suffix Array is tight, but much larger than genome
 - BWT is a reversible permutation of the genome based on the suffix array
 - Core index for Bowtie (Langmead *et al.*, 2009) and most recent short read mapping applications: BWA, SOAP, BLASR, etc...



BWT(Reference)





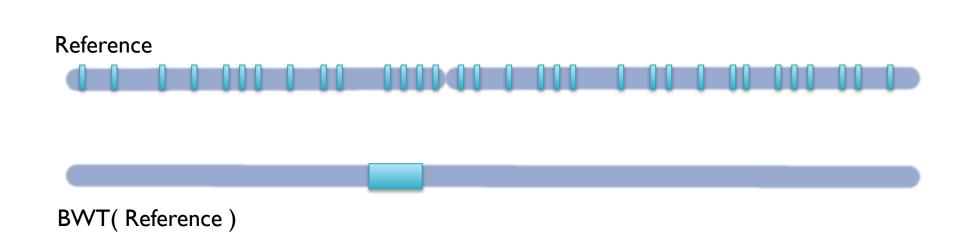
BWT(Reference)



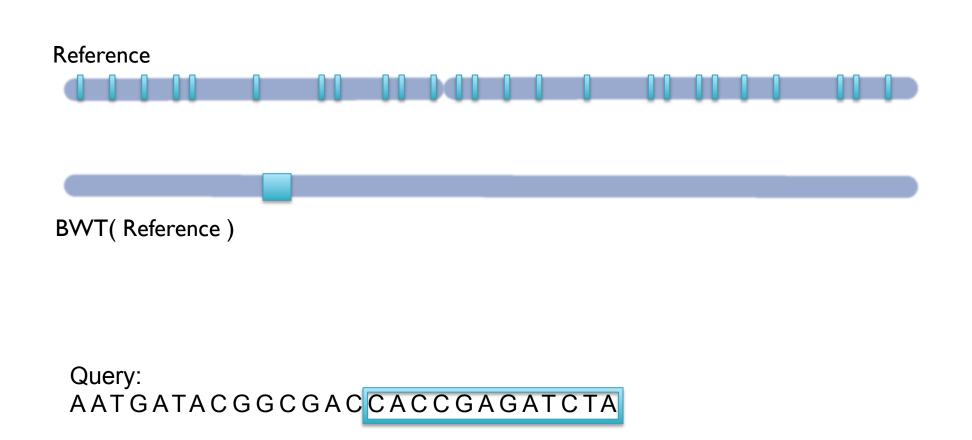


BWT(Reference)

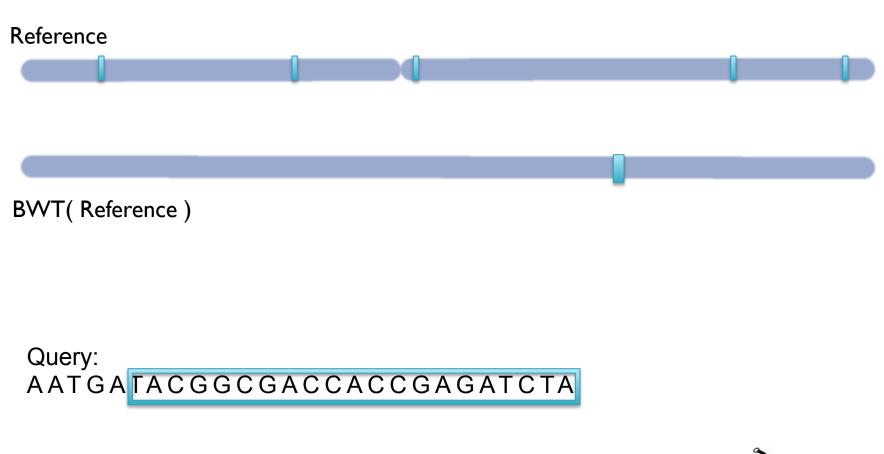




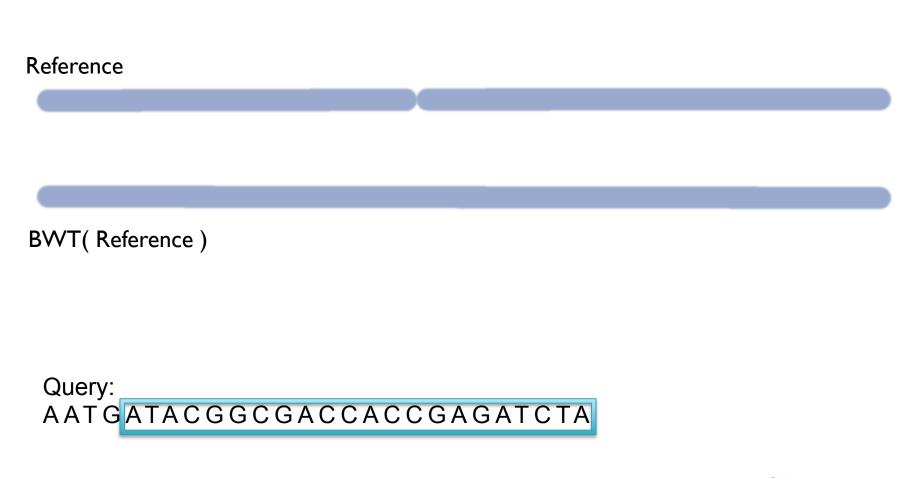




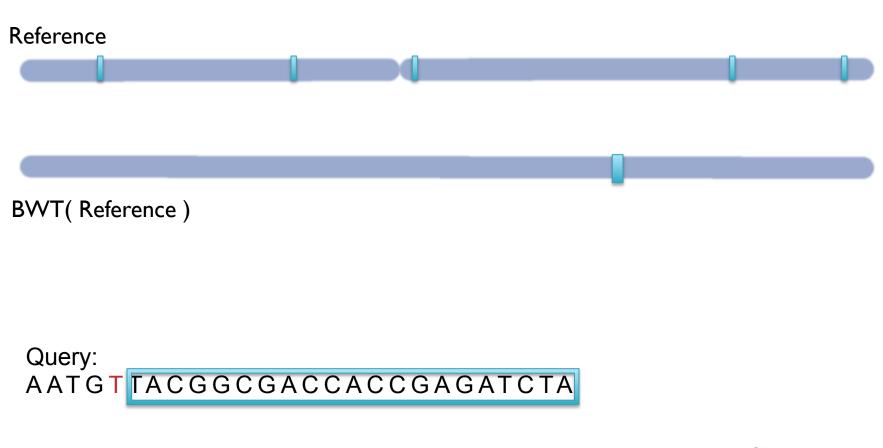




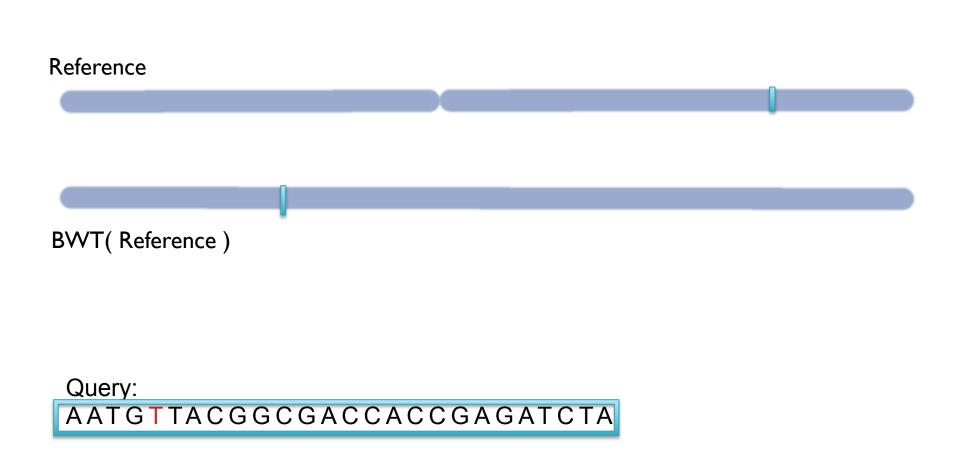














BWT Short Read Mapping

- Seed-and-extend search of the BWT
 - I. If we fail to reach the end, back-track and resume search
 - 2. The beginning of the read is used as high confidence seed
 - 3. BWT enables searching for good end-to-end matches entirely in RAM
 - I. 100s of times faster than competing approaches
- Report the "best" n alignments
 - I. Best = smallest hamming distance, possibly weighted by QV
 - 2. Some reads will have millions of equally good mapping positions
 - 3. If reads are paired, try to find mapping that satisfies both

Algorithms Summary

- Algorithms choreograph the dance of data inside the machine
 - Algorithms add provable precision to your method
 - A smarter algorithm can solve the same problem with much less work
- Techniques
 - Analysis: Characterize performance, correctness
 - Modeling: Characterize what you expect to see
 - Binary search: Fast lookup in any sorted list
 - Divide-and-conquer: Split a hard problem into an easier problem
 - Recursion: Solve a problem using a function of itself
 - Indexing: Focus on just the important parts
 - Seed-and-extend: Anchor the problem using a portion of it

"Think Harder & Compute Less" Dan Gusfield



http://schatzlab.cshl.edu